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Inner bremsstrahlung in electromagnetic zero–zero transitions in nuclei

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Abstract. The general form of the final state distribution is given for inner bremsstrahlung by the electron pair produced in 0–0 nuclear transitions with no parity change. The expression for the resultant photon energy spectrum is given and evaluated numerically for the transition in ^{16}O.

Here we wish to report the results of a calculation of the process of internal bremsstrahlung by the electron pair produced in electromagnetic zero–zero transitions in nuclei. Our results are complementary to those of Dalitz (1951) who gives a fairly full discussion of the problem. As the method is standard, we shall content ourselves here with a brief statement of the basis of the calculation and of the results, with particular reference to ^{16}O.

The process which we are considering is

$$A^* \rightarrow A + e^+ + e^- + \gamma \tag{1}$$

where A^* and A are nuclear levels of zero angular momentum and the same parity separated by an energy Δ; we shall be particularly concerned with the case of the 6·06 MeV and ground states of ^{16}O. The process can be represented as the sum of two diagrams, shown in figure 1. The interaction at the electron vertex is given by standard

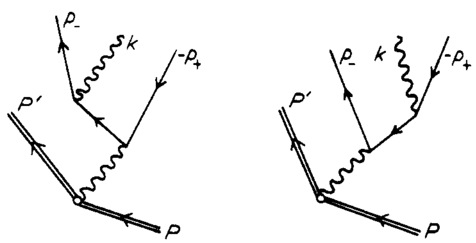

Figure 1.

electrodynamics as $e\bar{\psi}\gamma^\mu\psi A_\mu$; that at the nuclear vertex can in principle be calculated from a knowledge of the nuclear structure, but in practice we may not have sufficient knowledge to do this. However, to a good approximation it depends on only one

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parameter which cancels out when we compute the branching ratio of (1) compared to the main decay process

$$A^* \rightarrow A + e^+ + e^-. \tag{2}$$

The argument is as follows. The nuclear interaction can be written as $A_\mu j_\mu(q^2)$, where

$$q^2 \equiv q_0^2 - q_3^2 = \Delta^2 - q_3^2 \tag{3}$$

is the virtual photon mass. The nuclear transition current j_μ is a function of q^2 and we know that it must vanish at $q^2 = 0$, because there is no transition Coulomb field. We therefore write

$$j_\mu(q^2) = j'_\mu(0)q^2 \tag{4}$$

ignoring further dependence on q^2. But the only independent four-momenta at the vertex are P_μ and q_μ so we may write

$$j'_\mu(0) = ea_0(2P_\mu + \alpha q_\mu). \tag{5}$$

Since current conservation requires $q^\mu j_\mu = 0$, we find

$$j_\mu(q^2) = q^2 ea_0\left(2P_\mu - \frac{2M\Delta}{q^2} q_\mu\right) \tag{6}$$

and the effective potential a_μ acting on the electron current is such that

$$\not{a} \equiv a_\mu \gamma^\mu = \frac{j_\mu \gamma^\mu}{2Mq^2} = ea_0\left(\gamma^0 - \frac{\Delta \not{q}}{q^2}\right). \tag{7}$$

We have ignored the recoil kinetic energy of the nucleus of mass M, since

$$q_3^2/2M\Delta \leqslant \Delta/2M \ll 1.$$

The pair creation process (2) has a matrix element

$$\mathscr{M}_{ts} = -e(\bar{u}_t(p_-)\not{a} v_s(p_+)) \tag{8}$$

and a total probability

$$\lambda_\mathrm{P} = \frac{4a_0^2 e^4}{(2\pi)^3} \int_m^{\Delta - m} \mathrm{d}E_+ \, \mathrm{d}E_- p_+ p_- (E_+ E_- - m^2) \delta(E_+ + E_- - \Delta). \tag{9}$$

The bremsstrahlung matrix element is

$$\mathscr{M}_{ts\pi} = -e^2\left[\bar{u}_t(p_-)\left(\not{\epsilon}_\pi \frac{1}{\not{p}_- + \not{k} - m} \not{a} - \not{a} \frac{1}{\not{p}_+ + \not{k} + m} \not{\epsilon}_\pi\right) v_s(p_+)\right] \tag{10}$$

giving an angular distribution

$$\mathrm{d}\lambda_\mathrm{B} = \sum_{t,s,\pi} |\mathscr{M}_{ts\pi}|^2 \frac{1}{8kE_+E_-} \frac{\mathrm{d}^3k \, \mathrm{d}^3p_+ \, \mathrm{d}^3p_- \, \mathrm{d}^3P'}{(2\pi)^8} \delta^4(p_+ + p_- + k + P' - P) \tag{11}$$

where we use the electron spinor normalization $\bar{u} \,.\, u = 2m$. Carrying out the spin and polarization sums and the trivial integrations we find, again neglecting the recoil kinetic energy,

$$\lambda_\mathrm{B} = \frac{a_0^2 e^6}{(2\pi)^6} \int T \cdot p_+ p_- k \delta(E_+ + E_- + k - \Delta) \, \mathrm{d}E_+ \, \mathrm{d}E_- \, \mathrm{d}k \, \mathrm{d}z_+ \, \mathrm{d}z_- \, \mathrm{d}\phi_{+-} \tag{12}$$

where T can be written

$$T = T_{+-} + T_{++} + T_{--} + T_{+} + T_{-}$$

with

$$T_{+-} = \frac{2}{(p_+ \,.\, k)(p_- \,.\, k)}[p_+ \cdot p_-(kE_+ + kE_- + 2E_+E_- - p_+ \cdot p_-) - m^2(p_+ \cdot p_- + k^2)]$$

$$T_{++} = \frac{-m^2}{(p_+ \,.\, k)^2}(2E_+E_- - p_+ \,.\, p_- + 2kE_- - k \,.\, p_- - m^2) \tag{13}$$

$$T_{+} = \frac{-1}{p_+ \cdot k}[2(E_+^2 - E_+E_- - kE_- + p_+ \cdot p_-) + k \,.\, p_- - m^2]$$

T_{--} and T_{-} are given by the exchange $(+ \leftrightarrow -)$ on T_{++} and T_{+}; four-vector scalar products are used in (13) so $p_+ \,.\, k \equiv E_+k - p_+kz_+$ etc. From these formulae the spectra for any experiments which do not involve polarization measurements can be calculated. Dalitz (1951) gives the spectrum in cos θ, the angle between the electron and positron; we shall give here the photon energy spectrum and the branching ratio for 'hard' photons.

Integrating over all variables except the photon energy k, the photon spectrum may be written in the form

$$\begin{aligned}\lambda_B(k) = \frac{4e^6a_0^2}{(2\pi)^5}\frac{1}{k}\int_m^{\Delta-m-k} & \mathrm{d}E_+\,\mathrm{d}E_-\delta(E_+ + E_- + k - \Delta) \\ & \times\left\{\ln\left(\frac{E_+ + p_+}{E_+ - p_+}\right)\left[-\frac{m^2}{4}(2k^2+m^2)\ln\left(\frac{E_- + p_-}{E_- - p_-}\right) - p_-m^2(2\Delta - k)\right.\right. \\ & \left. + E_-p_-(k^2+3m^2) + 2E_+E_-p_-(k+E_+)\right] \\ & \left. - p_+p_-(5E_+E_- - 2kE_- + 4k(\Delta - k) - 4m^2)\right\}.\end{aligned} \tag{14}$$

The joint distribution in the two electron energy variables, E_+ and E_-, is obtained by symmetrizing the integrand in (14), that is by averaging it with the expression found by interchanging the subscripts + and − ; the photon energy k can be eliminated by using the δ-function.

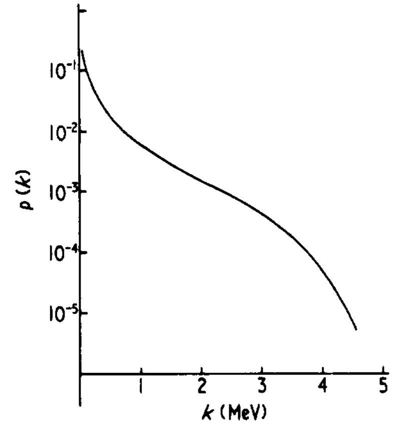
We have carried out the integration over the electron energies numerically for the transition in ^{16}O and the photon spectrum is shown in figure 2. The quantity plotted, $\rho(k)$, is the branching ratio per MeV

$$\rho(k) = \frac{\lambda_B(k)}{\lambda_P} \tag{15}$$

where λ_P is given by equation (9).

Because of the infrared divergence the total branching ratio is only defined for photons above a specified energy, $k_{\min}$; it can be well represented to 5% accuracy for $k_{\min} < 0{\cdot}1$ MeV, by

$$\rho = \int_{k_{\min}}^{5\cdot 04} \rho(k)\,\mathrm{d}k = 2{\cdot}5\left[10^{-2} + \lg\left(\frac{0{\cdot}1 \text{ keV}}{k_{\min}}\right)\right]. \tag{16}$$



k(MeV)	$\rho(k)$
0·5	0·01825
1·0	0·00701
1·5	0·00345
2·0	0·00181
2·5	0·00093
3·0	0·00045
3·5	0·00018
4·0	0·00006
4·5	0·00001

Figure 2. The photon energy spectrum.

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